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Quasi Instantaneous Estimation of Time Evolving ″Resonant Load″ Parameters: a Novel Approach based on the Hilbert Transform

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ABSTRACT

This paper presents a novel solution which is based on the trigonometric properties of band-limited signals represented in their analytical forms. Thus, the quasi-instantaneous estimation of the ″Dynamic Resonant Load Parameters″ takes **two Hilbert transformers and a few basic arithmetic functions**. This has the added advantage of producing an easily implementable solution with off-the shelf µcontrollers. The first series of computer simulations as well as Real-Time test have fully confirmed the theoretical results with very good global performances.

Keywords

″Short-Time″ Impedance Parameters Estimation – Hilbert Transformer - Real Time Implementation.

1. INTRODUCTION

Quasi instantaneous estimation of time evolving load parameters can serve for many applications. This is particularly true in non-stationary processes involving resonance where specific conditions must be permanently met in order to satisfy some standard operation criteria (e.g. efficiency, maximum peak current….). Processes with fast changing load conditions (e.g. high power ultrasound based processes) can greatly benefit from ″real-time″ load parameters estimations. Recent work [1,2,3] has clearly demonstrated that the Hilbert Transform (HT) has potentials often overlooked by the practitioners in the past. Moreover, the HT basic theory and their most important properties are well understood and increasingly gaining the attention among the engineering community. Furthermore, satisfactory compact IIR alternatives to the rather cumbersome FIR Hilbert transformer implementation have been presented [3,4]. Without much time devoted to optimization, the preliminary simulations as well as ″real-time″ tests (″Speedy-33″ TI VC33 based DSP and Hyperception VAB software) confirm the theoretical results. Some parts of the implementation aspects used in this design are based on previous work done by the author and developments made in a recent diploma work [5]. The most important characteristics are the followings:

- Frequency range of operation: 16 kHz → 25 kHz
- Clock frequency: 320 kHz
- Estimated load parameters:
  - Ratio of Voltage Envelope to Current Envelope:
    ″Short-Time Magnitude of the Impedance Z_{Load}″
  - Voltage and Current Phase Difference:
    ″Short-Time Argument of the Impedance Z_{Load}″
- Load parameters estimations group delay: 1/8th of the signal period (i.e. 6.25 µs at 20 kHz)
- Detectability threshold of load variation at resonance:

  0.5 % of active load value
- Detectability threshold of load phase changes at or near resonance: 0.5 degree.
- Phase estimation error due to changes of the ”Short-Time″ magnitude of Z_{Load}: less than 1 degree
- ″Short-Time″ argument of Z_{Load} estimation RMS error with band-limited Gaussian noise (16 to 25 kHz):

  Input SNR = 20dB ($SNR_{voltage} = SNR_{current}$) ➔ 6^0 (RMS)
- ″Short-Time″ magnitude of Z_{Load} estimation SNR with band-limited Gaussian noise (16 to 25 kHz):

  $SNR_{voltage} = SNR_{current} = 20$ dB ➔ $SNR_{Magnitude} \approx 20$ dB
- No bias in the magnitude or the phase estimations for input SNR above 10 dB
- No ″Arctan″ or other complex arithmetic functions required
- No potentially unstable operations and/or algorithms
- Easily adaptable to other frequencies of operation

This paper is organized as follows. The basic theoretical analysis is given in Section 2. Details of the proposed solution are presented in Section 3. Simulations results are shown in Section 4. Finally, the last Section summarizes the advantages of this design and suggests potential developments.

2. BASIC THEORETICAL ANALYSIS

Fig. 1 shows the basic bloc diagram that will be considered in this paper. By ″Resonant Load″ is assumed that the global quality factor will be at least larger or equal to 4.

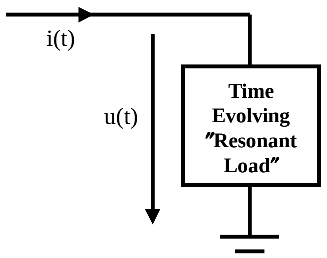

Figure 1: Basic structure bloc diagram

Let i(t) and u(t) be expressed as follows:

$$i(t) = \hat{I}(t)\ \cos(2\cdot\pi\cdot fo(t)\cdot t + \phi_i(t)) \quad (1)$$

$$u(t) = \hat{U}(t)\ \cos(2\cdot\pi\cdot fo(t)\cdot t + \phi_u(t)) \quad (2)$$

With:

$\hat{I}(t)$: Instantaneous current envelope

$\phi_i(t)$: Instantaneous current phase

$\hat{U}(t)$: Instantaneous voltage envelope

$\phi_u(t)$: Instantaneous voltage phase

fo(t): Instantaneous driving signal frequency

Note: All these instantaneous parameters are low-passed processes with cut-off frequencies well below fo(t)″.

Then, i(t) and u(t) can be represented in their respective analytical forms:

$$i(t) \Rightarrow i_{analytic}(t) = i(t) + j\cdot\tilde{i}(t) \quad (3)$$

$$u(t) \Rightarrow u_{analytic}(t) = u(t) + j\cdot\tilde{u}(t) \quad (4)$$

With:

$$\tilde{i}(t) = \hat{I}(t)\ \sin(2\cdot\pi\cdot fo(t)\cdot t + \phi_i(t)) \quad (5)$$

$$\tilde{u}(t) = \hat{U}(t)\ \sin(2\cdot\pi\cdot fo(t)\cdot t + \phi_u(t)) \quad (6)$$

From trigonometric properties, the following relationships can be easily derived:

$$\sin\left(\phi_u(t) - \phi_i(t)\right) = \frac{i(t)\cdot\tilde{u}(t) - \tilde{i}(t)\cdot u(t)}{\sqrt{i(t)^2 + \tilde{i}(t)^2}\cdot\sqrt{u(t)^2 + \tilde{u}(t)^2}} \quad (7)$$

$$M_{stZ}(t) = \frac{\sqrt{u(t)^2 + \tilde{u}(t)^2}}{\sqrt{i(t)^2 + \tilde{i}(t)^2}} \quad (8)$$

<u>Estimated Load Parameters:</u> **<u>New Definitions</u>**

(7) ➔ *Voltage and Current Phase Difference:* $\boldsymbol{\phi_{stZ}(t)}$

i.e. **″Short-Time″ Argument of the Impedance**

(8) ➔ *Ratio of Voltage Envelope to Current Envelope*: $\mathbf{M_{stZ}(t)}$

i.e. **″Short Time″ Magnitude of the Impedance″**

Thus, $\phi_{stZ}(t)$ and $M_{stZ}(t)$ can be estimated in ″Real-Time″ with a minimum of basic arithmetic functions.

3. HILBERT TRANSFORMER

3.1 Band-Limited Hilbert Transformer

A ″Band-Limited″ Hilbert transformer (BL-HT) is realized with one second-order all-pass filters and two unit delay elements. This solution has the double advantage of simplicity and very low group delay. Figure 2 shows the structure.

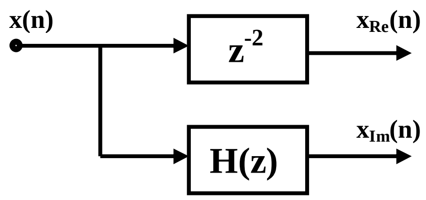

Figure 2: Band-Limited Hilbert Transformer

With:

x(n): Input, $x_{analytic}(n) = x_{Re}(n) + j\cdot x_{Im}(n)$ (9)

$$H(z) = \frac{-0.713 + z^{-2}}{1 - 0.713\cdot z^{-2}} \quad (10)$$

3.2 BL-HT Characteristics

An ideal HT has always 90^0 of phase difference between its ″In-Phase″ and ″Quadrature″ outputs, i.e. between $x_{Re}(n)$ and $x_{Im}(n)$ respectively. Moreover, both output amplitude transfer function are to be constant and independent of the frequency. This is naturally the case with the structure shown on Figure 1.

Summary of the BL-HT main characteristics:

- Sampling Rate: 320 kHz
- Frequency Range: 16 kHz → 25 kHz.
- Absolute phase error: less than 1^0
- Group delay: 6.25µs (constant)

4. COMPUTER SIMULATION

In this section, most of the characteristics listed in the introduction are demonstrated by computer simulation. The *″Time-Evolving″* load considered is shown on Figure 3. It is an R-L-C parallel structure where $R_L(t)$ represent the ″Active″ part of the load and L(t) controls its resonance frequency.

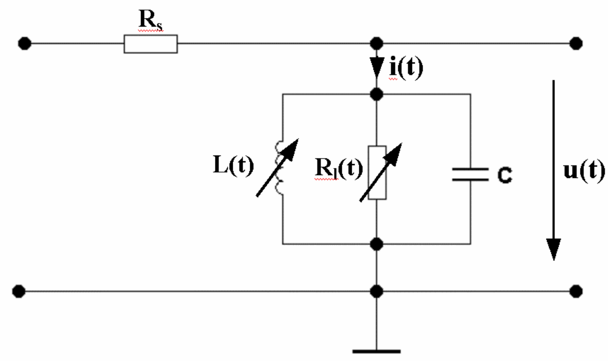

Figure 3: Time evolving resonant load

4.1 Multi-Parameters: general case

A first example is shown on Figure 4. It is purposely made somehow extreme in order to illustrate the very good performances of both estimators.

Variable parameters: Driving signal frequency
Active part of the load
Load resonance frequency

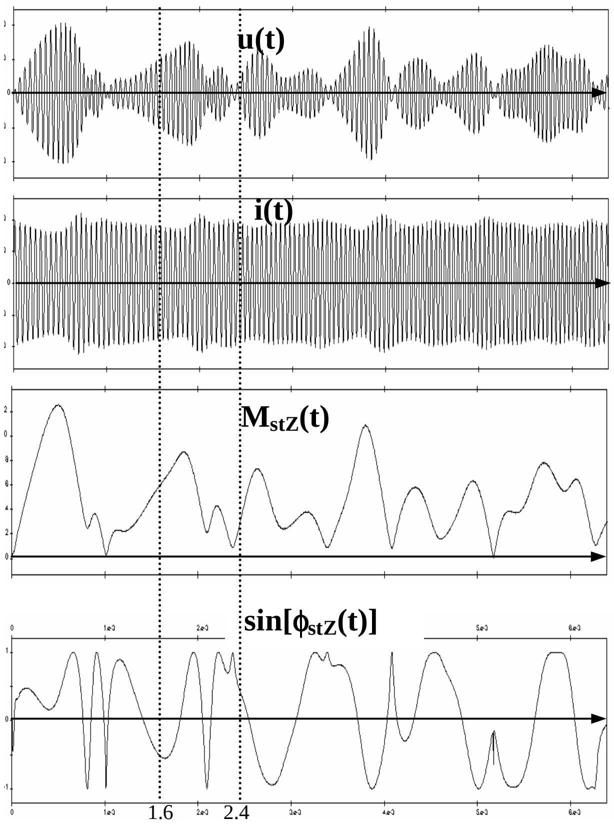

Figure 4: General case

Zoom of one small section (1.6 → 2.4ms) of Fig. 4:

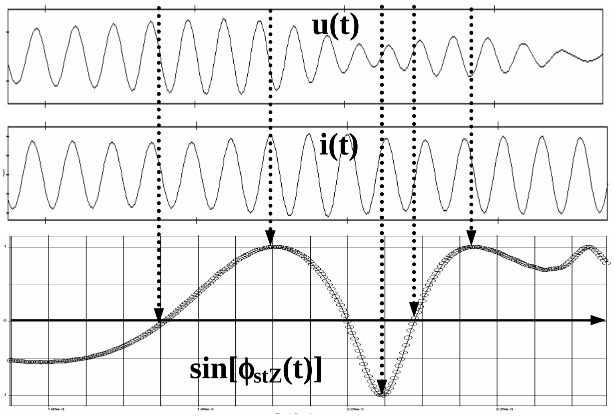

Figure 5: Fast *u(t)-i(t)* phase difference changes

This figure already confirms that the estimator ***sin[ϕ_{stZ}(t)]*** ″tracks″ extremely well *u(t)-i(t)* changes in phase difference.

4.2 ″Active″ part of the load: R_L(t)

Variable parameters: Driving signal frequency
Active part of the load

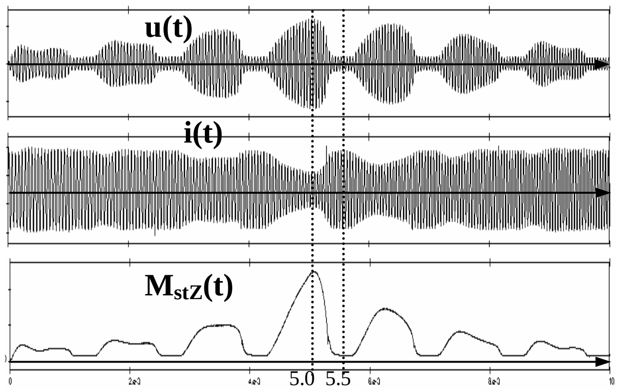

Figure 6: R(t) variation

Figure 7 (zoom of the 5 to 5.5 ms segment from Fig. 6) gives an idea of how fast the ″Short-Time″ magnitude of the impedance is determined..

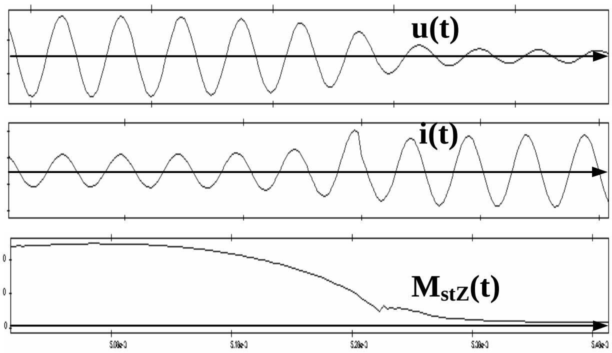

Figure 7: R(t) variation; zoom: 5.0 → 5.5ms

4.3 Estimators Group Delays

The tests are conducted with i(t) and u(t) as given by (1) and (2) respectively.

a) $M_{stZ}(t)$

Variable parameters: $\hat{U}(t)$
$\hat{I}(t)$
$\phi_u(t) - \phi_i(t)$: $-\pi/4 \rightarrow +\pi/4$

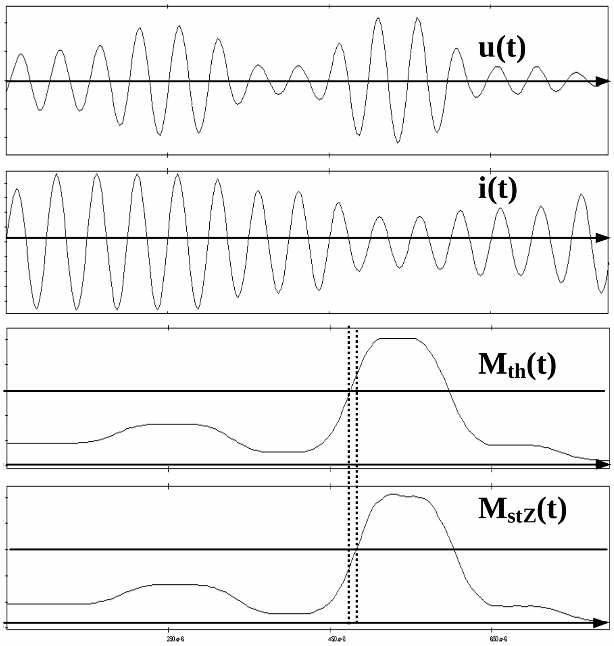

Figure 8 : Fast load impedance magnitude variation

This simulation confirms the ″6.25µs″ of group delay determined from BL-HT transfer function. $M_{th}(t)$ is the time varying theoretical load impedance magnitude.

b) $\sin[\phi_{stZ}(t)]$

Variable parameters: $\hat{U}(t)$
$\hat{I}(t)$
$\phi_u(t)$
$\phi_i(t)$

In Figure 9, the simulation of fast phase differences between u(t) and i(t) confirms that both parameter estimators have the same group delay (6.25µs), which is exclusively determined by the BL-HT transfer function. $\sin[\phi_{th}(t)]$ is the theoretical u(t)-i(t) phase difference. It is worth mentioning that the estimators are independent from each other, the only limitation being the Hilbert transformer characteristics (i.e. amplitude and phase response, group delay).

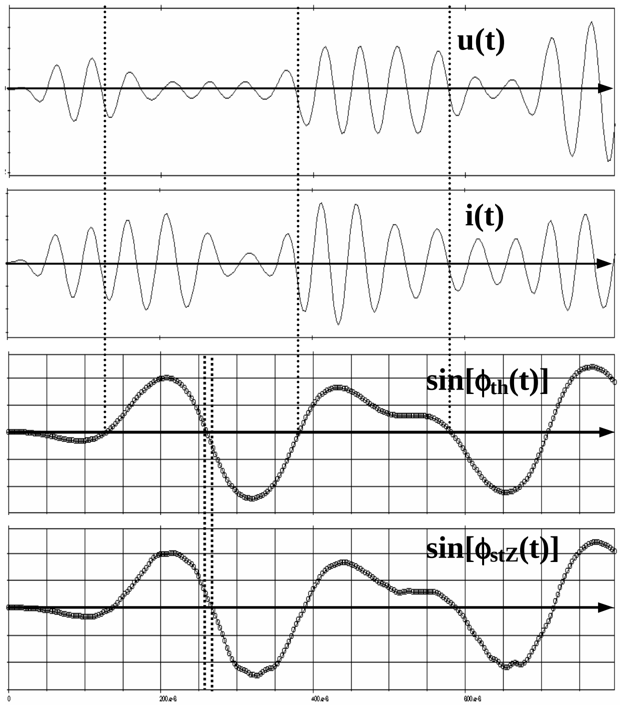

Figure 9 : Fast u(t)-i(t) phase difference variation

4.4 Load Variation Detectability Threshold

It is often useful to know how small a dynamically evolving load characteristic change can be reliably detected. With the next few examples we will show some limits that are inherently linked to the sampling rate as well as the BL-HT imperfections.

#1: Detectability threshold of R_L variation

Variable parameters: $R_L(t)$: ± 0.25 % variation
Driving signal frequency
(19.3 kHz → 21.3 kHz)

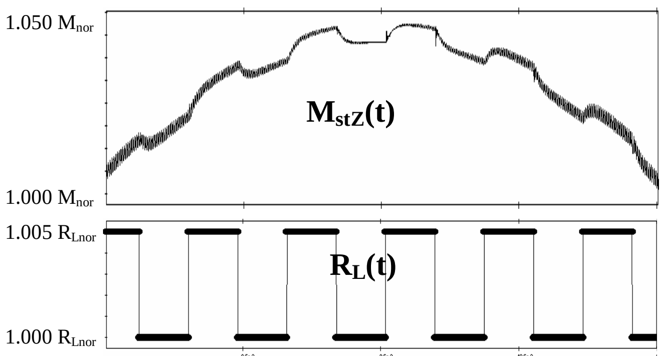

Figure 10: Small load variation around resonance

M_{nor}: normalized module

R_{Lnor}: normalized active part of the load

#2: Detectability threshold of load phase variation

Variable parameters: L(t) : ± 0.10 % variation
Driving signal frequency
(19.6 kHz → 19.7 kHz)

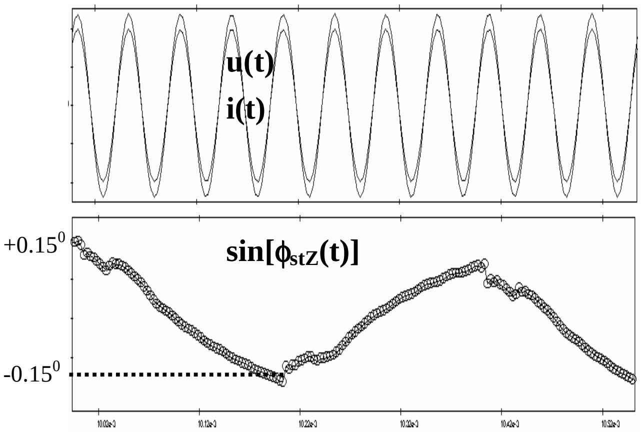

Figure 10: Very small u(t)-i(t) phase difference changes

This Figure 10 shows that very small u(t)-i(t) phase difference changes can still be detected and quantified. Notice that even a careful observation of u(t) and i(t) can hardly pin-points these tiny phase changes! This opens new research fields and potential innovation opportunities linked to the detection of ″Minor Changes of Process Parameters″ (e.g. in non-destructive testing).

4.5 Band-Limited Gaussian Noise

In the majority of ″Real Word Situation″, some noise will almost inevitably corrupt u(t) and/or i(t). Computer simulations has given the following results:

#1. ″Short-Time″ magnitude estimation SNR with band-limited Gaussian noise (16 to 25 kHz)

$SNR_{voltage} = SNR_{current}$ = 10 dB → $SNR_{Magnitude} \approx$ 5 dB
$SNR_{voltage} = SNR_{current}$ = 20 dB → $SNR_{Magnitude} \approx$ 20 dB
$SNR_{voltage} = SNR_{current}$ = 40 dB → $SNR_{Magnitude} \approx$ 40 dB

In Fig. 11, it can be seen how $M_{stZ}(t)$ approximate $M_{th}(t)$. Note that no filtering has been introduced yet; thus if some additional delay is acceptable, SNR as low as 10dB ($SNR_{voltage}= SNR_{current}$) could give useful approximation of the load magnitude.

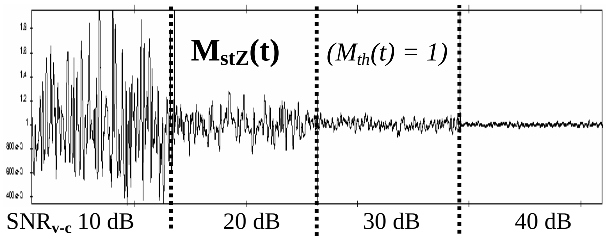

Figure 11: Identical SNR on both u(t) and i(t)

#2. ″Short-Time″ phase estimation RMS error with band-limited Gaussian noise (16 to 25 kHz)

Input SNR = 10dB ($SNR_{voltage} = SNR_{current}$) → 18^0 (RMS)
Input SNR = 20dB ($SNR_{voltage} = SNR_{current}$) → 6^0 (RMS)
Input SNR = 40dB ($SNR_{voltage} = SNR_{current}$) → 0.6^0 (RMS)

The next figure shows sin[ϕ(t)] estimation difference between ″SNR = 60 dB″ and ″SNR = 20 dB″.

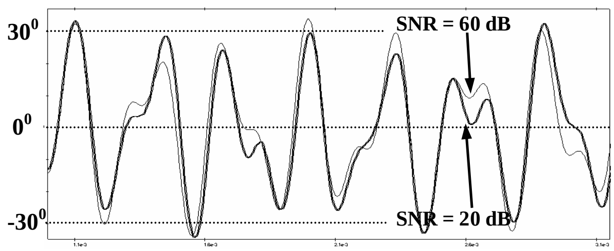

Fig. 12: sin[ϕ_{stZ}(t)] with identical SNR on both u(t) and i(t)

At this point it is worth noting that there is no significant bias in the magnitude or the phase estimations for input SNR above 10 dB. Moreover, preliminary mathematical analysis made in the case where the interferences are pure sine-waves [5] confirm the above results.

4.6 Impulsive Noise

The proposed new scheme does not appear overly sensitive to impulsive noise. This is best illustrated by the following examples:

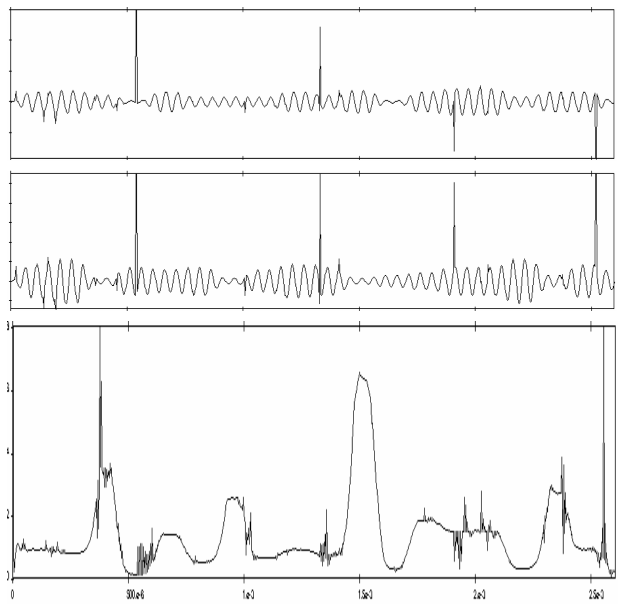

Figure 13: Consequences of impulsive noise on M_{stZ}(t)

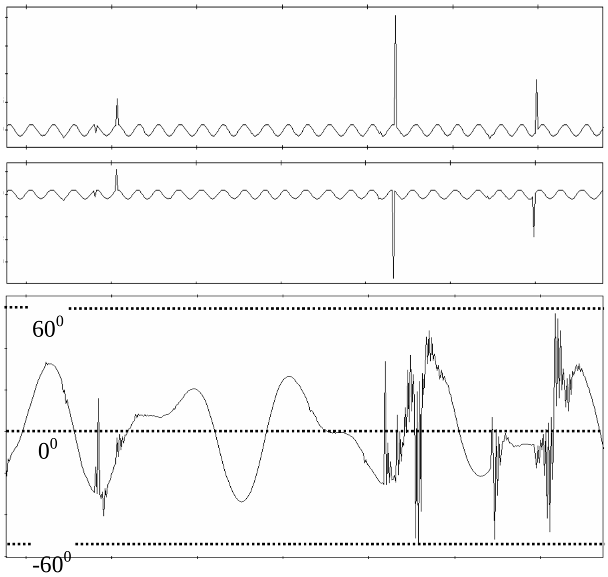

Fig. 14: Consequences of impulsive noise on sin[ϕ_{stZ}(t)]

Careful analysis show that sin[ϕ_{stZ}(t)] response to impulsive noise does not extend significantly beyond 28 samples (i.e. less than two signal cycles). Thus, median filtering techniques (on both voltage and current inputs) and/or averaging can be added as a protection against such undesirable events.

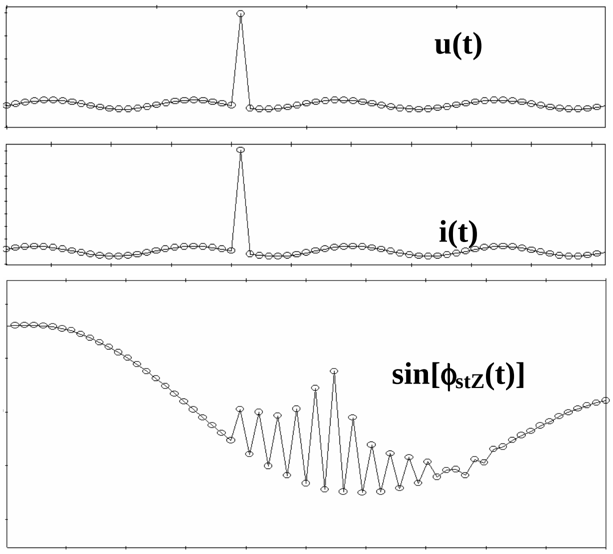

Figure 15: sin[ϕ_{stZ}(t)] response to impulsive noise

5. CONCLUSIONS

This paper describes a new ″Quasi-Instantaneous Estimator of Resonant Load Parameters″ based on two Hilbert Transformers and a few basic arithmetic functions. The first series of computer simulations as well as ″Real-Time″ experiments are in good agreement with the theoretical analysis. Moreover, the many tests conducted have proven the ″robustness″ of this solution at the same time as producing extremely fast and accurate results.

Our latest research work indicates that the same set of blocs can also be used to determine the ″**Short-Time Active Power of Time Evolving Resonant Loads**″. While these interesting properties are currently been thoroughly analyzed, they are already proving to be very useful in non-stationary high-power systems. This can also dramatically enhance the global performances (e.g. efficiency) of some industrial processes. In addition, we can also consider implementing these original schemes to the design of low-cost band-limited network analyzers (e.g. characterization of ultrasonic transducers), to new families of ″Ultra Fast Power Metering Systems″ etc....

6. REFERENCES

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7. BIOGRAPHY

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